Homework 39: multivariable differentiation

(1) The Ideal Gas Law states that $PV = nRT$, where P is pressure of an ideal gas, V is volume, T is temperature, n is the number of molecules in terms of moles, and R is the ideal gas constant. Write the volume V as a function dependent on the two independent variables pressure P and temperature T. Treat n and R as constants. Find the partial derivatives V_T and V_P.

(2) (a) Find both partial derivatives for $h(x, y) = 2(xy - y)^3$. Check your answers with *Mathematica*, print and attach.
   (b) Find both partial derivatives for

$$g(x, y) = \frac{xy^5 - 4y^2 + 6x^4y^7}{\sin x}.$$

   You may, of course, check your answers (here and below) with *Mathematica*, but there is no need to print and attach.
   (c) Find $\frac{\partial P}{\partial A}$ and $\frac{\partial P}{\partial S}$ if $P = 4M^2e^{2S-A^2+AS+5M}$.

(3) (a) Using the table of wind chill values from `https://www.weather.gov/safety/cold-wind-chill-chart`, numerically estimate $C_t(15, 30)$ and $C_v(15, 30)$ where the function is represented by $C(t, v)$ and t is temperature in degrees Fahrenheit and v is wind speed in mph. Interpret your results in complete sentences in a way that an intelligent person who did not take calculus can understand. (In particular, this person knows the meaning of English words such as "increase" and "rate" but not mathematical jargon such as "derivative" and doesn't like hearing math-sounding phrases like "rate of change with respect to.")
   (b) Using $C(t, v) = 35.74 + 0.6215t - 35.75v^{0.16} + 0.4275tv^{0.16}$, find C_t and C_v in general and then for the point $(15, 30)$. Compare your results with part (a).

(4) In order to treat patients, doctors sometimes need to know the surface area of a person's body. Since this is difficult to measure directly, an estimate is usually made from the person's height and weight. For a child of height h inches and weight w pounds, the surface area in square inches can be approximated by

$$S(h, w) = 8.52h^{0.35}w^{0.54}.$$

(Source: *Multivariable Calculus* by McCallum *et al.*)
   (a) Find $\frac{\partial S}{\partial h}$ at $(40, 50)$ and interpret this quantity in practical terms. Give units with your numerical estimate and show 4 significant digits in your result.
   (b) Repeat for $\frac{\partial S}{\partial w}$ as above.
   (c) Which of the two partial derivatives you found is larger? Why might you have expected this?