Math131A Set 1

Due at the lecture on Monday, **July 1, 2013**.

Collaboration is encouraged, as long as you **write your own solutions** and **write down the name of your collaborators**.

1. Natural Numbers $\mathbb{N}$

1.1. Prove

$$23 + 29 + 35 + \ldots + (6n - 1) = 3n^2 + 2n - 33$$

for all natural numbers $n \in \mathbb{N}$. [You may start with $n \geq 4$.]

1.2.

(a) Decide for which natural numbers $n \in \mathbb{N}$ the inequality $3^n > n^3$ is true.
(b) Prove that your claim is correct by mathematical induction.

1.3. **Commutativity of addition.** Using the Peano axioms, prove that

$$m + n = n + m \text{ for all } n, m \in \mathbb{N}.$$

For $n \in \mathbb{N}$, let P_n be the statement $m + n = n + m$ for all $m \in \mathbb{N}$.

(a) Prove that P_0 is true using mathematical induction on m, *i.e.*, prove that $m + 0 = 0 + m$ for all m.
(b) (Optional) Prove that P_1 is true.
(c) Prove that if P_n is true, then so is P_{n+1}.

[You may assume the associativity of addition: $(a + b) + c = a + (b + c)$ for $a, b, c \in \mathbb{N}$.]

2. Rational Numbers $\mathbb{Q}$

2.1. Prove that the following are not rational numbers:

(a) $\alpha = \sqrt{2} - \sqrt{3}$
(b) $\varphi = \frac{1+\sqrt{5}}{2}$

3. Ordered Fields

Let $(F, 0, 1, +, \cdot, \leq)$ be an ordered field.

3.1. **Addition of inequalities.** Suppose $a \leq b$, $c \leq d$ in F. Prove that $a + c \leq b + d$.

3.2. **Generalized triangle inequality.** Prove that

$$|a_1 + a_2 + \ldots + a_n| \leq |a_1| + |a_2| + \ldots + |a_n|$$

for n elements $a_1, a_2, \ldots, a_n \in F$.

4. Real Numbers $\mathbb{R}$

Do parts (b), (k), and (v) of exercises 4.1, 4.3, and 4.4 in the book. [For (k), start with $n = 1$.]

Do exercises 4.7 and 4.8 in the book.