Math131A Set 2

Due at the lecture on Monday, **July 8, 2013**.

Collaboration is encouraged, as long as you **write your own solutions** and **write down the name of your collaborators**.

7. Sequences

Do exercise 7.4 in Ross.

8. Proofs of limits of sequences

8.1. Let (s_n) be a sequence of nonnegative real numbers converging to s.

(a) Prove that $s \geq 0$.
(b) Suppose $s = 0$, prove that $\lim \sqrt{s_n} = 0$.

8.2. For each sequence, *use the definition of the limit* to prove it converges to some real number, or prove that it diverges. **Do not use theorems about limits from Section 9.**

(a) $\frac{2n-5}{6n-5}$
(b) $\frac{n^2+3}{n-4}$
(c) $\frac{7n+2}{2n^2+42}$
(d) $\sin(n\pi)$
(e) $\cos(n\pi)$
(f) $\frac{1}{n}\sin(n^2+2n+1)$
(g) $\sqrt{n^2+4n}-n$

8.3. Do part (a) of exercises 8.5, 8.6, and 8.9 in Ross.

9. Theorem of limits of sequences

9.1. For each sequence, *use the theorems in Section 9* to prove it converges to some real number, or prove that it diverges. [Note that the first three are the same as those in 8.2 above.]

(a) $\frac{2n-5}{6n-5}$
(b) $\frac{n^2+3}{n-4}$
(c) $\frac{7n+2}{2n^2+42}$
(d) $\frac{82n^4+3n^3-200n^2+1}{17n^4-7n^2}$

9.2. Let $a_0 = 7$ and let $a_{n+1} = \sqrt{a_n + 3}$ for $n \in \mathbb{N}$. Given that (a_n) converges, calculate its limit.

9.3. **Comparison.** Suppose that $a_n \leq b_n$ *eventually*, i.e., there exists N such that for all $n > N$, $a_n \leq b_n$. Prove that if $\lim a_n$ and $\lim b_n$ exist, then $\lim a_n \leq \lim b_n$.

9.4. Prove that $\lim n^5 = +\infty$ only using the definition of the limit diverging to $+\infty$.

9.5. **Series.** Calculate $\lim_{n\to\infty}(1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n})$.

10. Monotone and Cauchy sequences

Do exercises 10.1, 10.6, 10.7, 10.8, and 10.10 in Ross.