Math131A Set 4

Due at the lecture on Monday, **July 22, 2013**.

Collaboration is encouraged, as long as you **write your own solutions** and **write down the name of your collaborators**.

17. Continuous functions

17.1.   Let $f(x) = x + 23$ and $g(x) = \sqrt{x}$, $x \geq 0$.
   (a) Find the domain of $f + g$, fg, $f \circ g$ and $g \circ f$.
   (b) Prove that $f \circ g \neq g \circ f$.
   (c) Show that $\sqrt{x + 23}$ is continuous on its domain.

17.2.   A function f is called a *rational function* if it can be written as the quotient $f = p/q$ of polynomial functions p and q, i.e., p is of the form $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ and similarly for q. Prove that rational functions are continuous. [Hint: Prove that polynomials are continuous first.]

17.3.   Prove the following functions are continuous or discontinuous at $x = a$ by using either the definition or Theorem 17.2 (the ε-δ definition).
   (a) $f(x) = 1/x^2$, $a = 7$.
   (b) $f(x) = x^3$, any $a \in \mathbb{R}$.
   (c) $f(x) = \cos(\frac{1}{x})$ for $x \neq 0$, $f(0) = 0$, $a = 0$.
   (d) $f(x) = x \cos(\frac{1}{x})$ for $x \neq 0$, $f(0) = 0$, $a = 0$.

17.4.   For $x \in \mathbb{Q}$ a rational number, write x in its reduced form: $x = p/q$, $p, q \in \mathbb{Z}$, $\gcd(p, q) = 1$, and $q > 0$. Define $f(x) = 1/q$ for each rational number $x \in \mathbb{Q}$. Otherwise, define $f(x) = 0$ for $x \in \mathbb{R} \backslash \mathbb{Q}$. Show that $f(x)$ is continuous at all irrational points $x \in \mathbb{R} \backslash \mathbb{Q}$ and discontinuous at all rational points $x \in \mathbb{Q}$.

18. Properties of continuous functions

Do exercises 18.4, 18.5, and 18.9 in Ross.

19. Uniform continuity

Do exercises 19.6, 19.7, and 19.9 in Ross.