Math 4707 Midterm 2 Practice Questions

Assume that all graphs are simple.

Problem 1. Let G be a graph on vertex set $[7]$.

(a) Could the vertices of G have degrees $1, 3, 4, 4, 5, 5, 5$?
(b) Could the vertices of G have degrees $0, 1, 3, 4, 4, 4, 6$?
(c) How many 2-regular graphs (every vertex has degree 2) on vertex set $[7]$ are there?

Problem 2. Suppose G is a graph with exactly two vertices u and v of odd degree. (G may have vertices of even degrees.) Prove or disprove the following.

(a) G is connected.
(b) There is a path from u to v.

Problem 3. Recall that there are n^{n-2} (labelled) trees on $[n]$.

(a) What is the number of trees on $[n]$ where 1 is a leaf?
(b) What is the number of trees on $[n]$ where 1 has degree 2?

Problem 4. Suppose G is a forest on 100 vertices and 70 edges. How many connected components could G have?

Problem 5. A *chord* of a cycle is an edge connecting two non-adjacent vertices of the cycle. Show that if every vertex of G has degree at least 4, then G has a cycle with two (or more) chords.

Problem 6. Let G be a connected graph on n vertices and n edges, $n \geq 3$.

(a) Show that G can be obtained from a tree with n vertices by adding a new edge.
(b) Show that the number of spanning trees of G is at least 3 and at most n.

Problem 7. An n-*wheel* is the graph obtained from an n-cycle by adding a new vertex that is adjacent to all n vertices of the cycle. (Thus an n-wheel has $n + 1$ vertices and $2n$ edges.)

(a) Find the number of perfect matchings of an n-wheel.
(b) Find the number of matchings of an n-wheel.

Problem 8. Show that a graph is bipartite if and only if it has no odd cycles.