Math 4990 Problem Set 4

Due Tuesday, Sep 30, 2014 in class

Please refer to previous problem sets for instructions, including but not limited to the collaboration policy.

Read the assignment carefully.

Assignment

Liberally peruse **pages 33–36** of [DO].

Problem 1. Recall these definitions from class:

A set $M \subseteq \mathbb{R}^n$ is **convex** if for every pair $x, y \in M$, the line segment from x to y lies in M.

For a set $S \subseteq \mathbb{R}^n$, the **convex hull** of S, denoted $\text{conv}(S)$, is the intersection of all convex sets that contain S.

Prove that the convex hull $\text{conv}(S)$ of any set S is convex.

(Do not assume that S lies in the plane. Do not use Theorem 2.2, as its proof relies on this exercise.)

Problem 2. Let $S \subset \mathbb{R}^n$ be a finite point set with at least four points. For $n = 2$, show that S can be partitioned into two sets A and B such that $\text{conv}(A)$ intersects $\text{conv}(B)$. (Do not use Helly theorem, as this fact is used in its proof.) Does the result hold for $n \geq 3$? (Provide justification.)

Problem 3. Let $S \subset \mathbb{R}^2$ be a finite point set in the plane. Show that $\text{conv}(S)$ is the convex polygon with the smallest perimeter that contains S.

Problem 4. Let $S \subset \mathbb{R}^3$ be a finite point set in space. Show that $\text{conv}(S)$ is the convex polyhedron with the smallest volume that contains S.

Problem 5. Let $P_1, \ldots, P_n \subset \mathbb{R}^2$ be rectangles whose sides are parallel to the x- and y-axes. Show that if every *two* of them intersect then they all intersect, *i.e.*, there is a point $z \in P_1, \ldots, P_n$.