Math 4990 Problem Set 6

Due Tuesday, Oct 14, 2014 in class

Errata

p.65, Unsolved Problem 11, "exponential number of ~~triangles~~ triangulations"

p.68, last line, "by our induction hypothesis establishes the theorem."

Assignment

Liberally peruse **pages 64–69, 98–102** of [DO].

[DO] Exercises 3.19, 3.20 (for $n \geq 4$), 4.4, and 4.5 ("simple" means at most a few sentences).

Problem 5. Let G be a triangulation graph and a, b, c three of its vertices. Show that G has a vertex v distinct from a, b, c such that the degree of v is at most five.

Note that this is a strengthening of Exercise 3.14 we used in class for the proof of Fáry theorem.

Problem 6. Recall that the number of triangulations of an $n + 2$-gon is the Catalan number C_n. For infinitely many values of n, construct a point set S with $n + 2$ points such that the number of triangulations of S is greater than C_n. (See Exercises 3.15 and 3.18.)

Note that "for infinitely many values of n" is a phrase mathematicians use when they want something more general than $n = 23$, say, but do not need it for every single value of n. For example, perhaps your construction works only for even n greater than 42, prime numbers, or n such that its proper positive integer divisors sum to itself. We refer to these as "infinite families of counterexamples."