CHAPTER

11 Review

11.1 Definitions in Chapter 11

11.1 Tree

11.2 Node

11.4 Root; Rooted Tree

11.5 Level

11.6 Parent; Child; Ancestor; Descendant; Sibling

11.7 Subtree

11.8 Leaf Node; Interior Node

11.9 Height; Balanced Trees

11.10 Ordered Rooted Tree

11.11 *m*-ary Tree; Binary Tree; Ternary Tree; Full Tree; Complete Tree; Maximal Complete Tree

11.17 External Nodes

11.21 Eccentricity; Center

11.22 Traversal

11.23 Preorder; Inorder; Postorder

11.24 Binary Search Tree

11.28 Binary Heap

11.29 Parse Tree

11.30 The Prefix Property

11.31 Well-formed

11.32 Spanning Tree

11.37 Minimal Spanning Tree

11.2 Sample Exam Questions

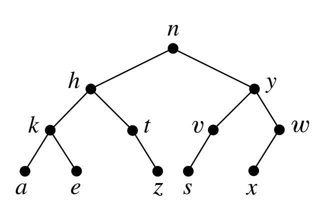
1. Prove that a tree with n vertices has $n - 1$ edges.

2. Let T be a maximal complete m-ary tree with height h.
   (a) How many nodes are at level j?
   (b) Write the number, n, of nodes in T as a simple expression in m and h.
   (c) Prove your answer to part (b).

3. Build a binary search tree using the following values (in left-to-right order).

$$d \quad f \quad c \quad a \quad b \quad t \quad w \quad r \quad s$$

4. Show the results of preorder, inorder, and postorder traversals for the following binary tree.



5. Use a tree to convert the infix expression `4^2 + 5*7` to postfix. Show the tree and the resulting postfix expression.

6. I am currently using the following encoding to store certain characters as binary strings on a computer.

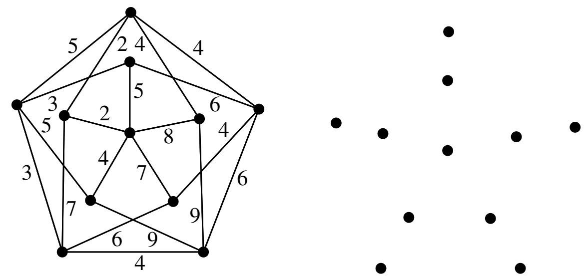
Character	Code
e	000
f	010
o	110
r	011
$\sqcup$	111

(a) Create a Huffman code that does a better (fewer bits) job of storing the following 10 character message.

error free

(b) Determine the total number of bits needed under each encoding.

7. Find a minimal spanning tree for the following graph. Briefly describe the algorithm you used. Indicate the total weight of your spanning tree.



11.3 Projects

Mathematics

1. Write a brief expository paper on backtracking algorithms. Provide one or two detailed examples that are different from any backtracking algorithms found in this book.

2. Write a brief expository paper on the connections between Cayley's formula and chemistry.

3. Write a paper illustrating the use of trees in decision theory. Discuss the role of Bayes's theorem.

4. Write a brief expository paper on Fibonacci trees. Define them, and prove a theorem about the number of vertices in the nth Fibonacci tree. What other theorems can you find or prove?

Computer Science

1. Write a program that uses a tree to convert infix expressions to postfix.

2. Write a program which implements the Huffman compression algorithm. The program should analyze the text to calculate the character frequencies before building any trees. Assume that only the 256 standard ASCII characters are permitted. The encoding tree will need to be stored with the compressed data to enable the message to be uncompressed.

3. Write a brief expository paper to compare and contrast DTDs and schemas.

4. Write a brief expository paper on the use of parse trees in compilers.

11.4 Solutions to Sample Exam Questions

1. This is Theorem 11.12.

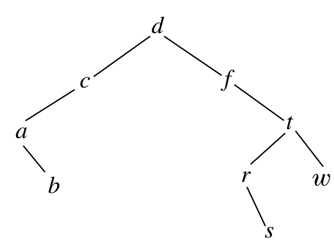
The proof is quite simple and elegant. Observe that every node except the root has exactly one adjacent edge on the unique path joining the node to the root. Every edge has two incident nodes. Identify the edge with the node that is farther from the root (determined by the respective levels). There is a one-to-one association between edges in the tree and non-root nodes. Since there are $n-1$ nodes that are not the root, there must be $n-1$ edges.

2. (a) There are m^j vertices at level j.

(b) $n = \sum_{j=0}^{h} m^j = \frac{m^{h+1}-1}{m-1}$.

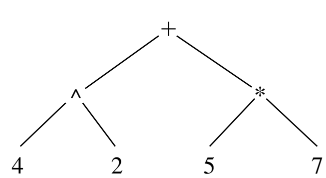
(c) At level 0 there is $m^0 = 1$ node. At the next level there will be $m^1 = m$ nodes, since every interior node in a full tree has m children. At level 2 there will be m^2 nodes since there are m nodes at level 1, each having m children. In general, there will be m^j nodes at level j. The total number of nodes will be $\sum_{i=0}^{h} m^i = \frac{m^{h+1}-1}{m-1}$.

3. The search tree that results is shown.



4. **Preorder** *n h k a e t z y v s w x*
Inorder *a k e h t z n s v y x w*
Postorder *a e k z t h s v x w y n*

5. The corresponding binary tree is shown.



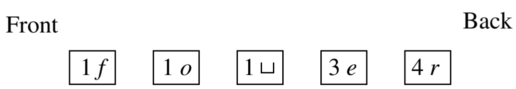
A postorder traversal yields 4 2 ^ 5 7 * +.

6. The character frequencies in the message are listed in the following table.

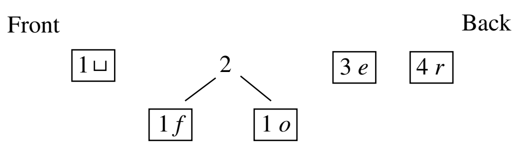
Character	Frequency
e	3
f	1
o	1
r	4
⊔	1

(a) The steps of the Huffman algorithm are shown below, without commentary.

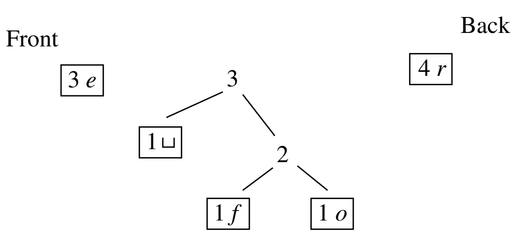
Step 1



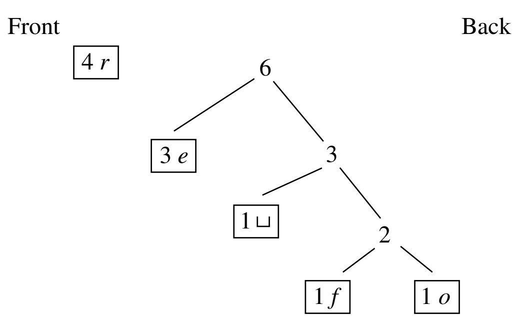
Step 2



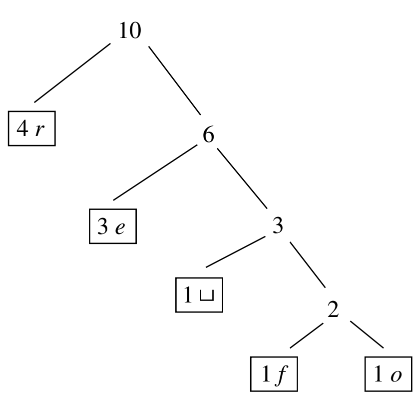
Step 3



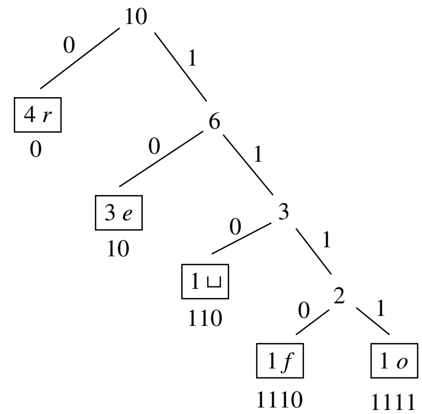
Step 4



Step 5



Step 6



The character encodings are shown in the next table.

Character	Code
e	10
f	1110
o	1111
r	0
⊔	110

(b) The original encoding takes $10 \cdot 3 = 30$ bits. The new encoding takes $3 \cdot 2 + 4 \cdot 1 + 1 \cdot 4 + 1 \cdot 4 + 1 \cdot 3 = 21$ bits (a 30% savings).

7. A minimal spanning tree is shown. It can be found with either Prim's algorithm or Kruskal's algorithm. There are multiple correct solutions, but all share the weight 2, 3, and 4 edges in common. They differ only in which weight 5 edge is chosen. The total weight is 35.