APPENDIX

G Writing Mathematics

The following suggestions will help you to submit properly written homework solutions. The suggestions are appropriate for both computational exercises and for proofs.

The goal of any writing is to clearly communicate ideas to another person. (That other person may even be your future self.) When you write for another person, you will need to include ideas that may be in your mind but omitted when you are writing a rough draft on scratch paper. If you keep your intended audience in mind, you will produce higher quality work. For a course in mathematics, the intended audience is usually your instructor or student grader. This implies that your task is to show that you thoroughly understand your solution. Consequently, you should routinely include more of the details.

1. **Use Sentences** The feature that best distinguishes between a properly written mathematical exposition and a piece of scratch paper is the use (or lack) of sentences. Properly written mathematics can be read in the same manner as properly written sentences in any other discipline. Sentences force a linear presentation of ideas. They provide the connections between the various mathematical expressions you use. This linearity will also keep you from handing in a page with randomly scattered computations with no connections. The sentences may contain both words and mathematical expressions. The following extract from a Quick Check solution illustrates these ideas.

   > Let n be odd. Then Definition 3.10 indicates that there does not exist an integer, k, such that $n = 2k$. That is, n is not divisible by 2. The Quotient–Remainder theorem asserts that n can be uniquely expressed in the form $n = 2q + r$, where r is an integer with $0 \leq r < 2$. Thus, $r \in \{0, 1\}$. Since n is not divisible by 2, the only admissible choice is $r = 1$. Thus, $n = 2q + 1$, with q an integer.

2. **Read out loud** The sentences you write should read well out loud. This will help you to avoid some common mistakes. Avoid sentences like:

   > Suppose the graph has n number of vertices.
   > The piggy bank contains n amount of coins.

   If you substitute an actual number for n (such as 4 or 6) and read these out loud they will sound wrong (because they *are* wrong). The variable n is already a numeric variable so it should be read just like an actual number. The correct versions are:

   > Suppose the graph has n vertices.
   > (Read this as: "Suppose the graph has en vertices".)
   > The piggy bank contains n coins.

3. **= is not a conjunction** The mathematical symbol = is an assertion that the expression on its left and the expression on its right are equal. Do not use it as a connection between steps in a series of calculations. Use words for this purpose.

   Here is an example that misuses the = symbol ($6 = \frac{3x}{3}$ is false):

$$\textbf{Incorrect!} \qquad 3x = 6 = \frac{3x}{3} = \frac{6}{3} = x = 2$$

   One proper way to write this is

   $3x = 6$. Dividing both sides by 3 leads to $\frac{3x}{3} = \frac{6}{3}$, which simplifies to $x = 2$.

4. **Do not Merge Steps** Suppose you need to calculate the final price for a \$20 item with 7% sales tax. One strategy is to first calculate the tax, then add the \$20. Here is an incorrect way to *write* this.

$$\textbf{Incorrect!} \qquad 20 \cdot 0.07 = 1.4 + 20 = \$21.4.$$

   The main problem (besides the magically-appearing dollar sign at the end) is that $20 \cdot 0.07 \neq 1.4 + 20$. The writer has taken the result of the multiplication (1.4) and merged directly into the addition step, creating a lie (since $1.4 \neq 21.4$). The calculations could be written as:

$$\$20 \cdot 0.07 = \$1.40 \quad \text{so the total price is} \quad \$1.40 + \$20 = \$21.40$$

5. **Use a vertical format for long calculations** If you have a long sequence of calculations, consider writing it vertically instead of horizontally. Line things up on some connecting symbol such as = or $\leq$. Whenever possible, make the vertical format read in a linear fashion. The first line will have expressions on both sides of the connecting symbol, but subsequent lines will only have expressions on the right. For example, the sequence of simplifications

$$2 \cdot \left(n \cdot \frac{n}{2} + \frac{n}{2}\right) = 2 \cdot \left(\frac{n^2}{2} + \frac{n}{2}\right) = 2 \cdot \left(\frac{n^2 + n}{2}\right) = n^2 + n$$

   can be written (notice the empty left-hand sides in rows 2 and 3):

$$\begin{aligned} 2 \cdot \left(n \cdot \frac{n}{2} + \frac{n}{2}\right) &= 2 \cdot \left(\frac{n^2}{2} + \frac{n}{2}\right) \\ &= 2 \cdot \left(\frac{n^2 + n}{2}\right) \\ &= n^2 + n \end{aligned}$$

6. **Avoid ambiguity** When in doubt, repeat a noun rather using unspecific words like "it" or "the". For example, in the sentences

   > Let G be a simple graph with $n \geq 2$ vertices that is not complete and let $\overline{G}$ be its complement. Then it must contain at least one edge.

   there is some ambiguity about whether "it" refers to G or to $\overline{G}$. The second sentence is better written as "Then $\overline{G}$ must contain at least one edge".

7. **Don't Neglect the Distributive and Associative Laws** There are many ways that students forget to use or write the distributive and associative laws correctly. Here are some examples of abuse, together with a correct version (in bold font).

$$3(3 + 4a_n) = 9 + 4a_n \qquad \mathbf{3(3 + 4a_n) = 9 + 12a_n}$$

$$\sum_{i=0}^{n} i + 2^i = \frac{n(n+1)}{2} + \frac{2^{n+1} - 1}{2 - 1} \qquad \sum_{\mathbf{i=0}}^{\mathbf{n}} \left(\mathbf{i} + \mathbf{2^i}\right) = \frac{\mathbf{n(n+1)}}{\mathbf{2}} + \frac{\mathbf{2^{n+1} - 1}}{\mathbf{2 - 1}}$$

$$a + b \cdot x = a \cdot x + b \cdot x \qquad \mathbf{(a + b) \cdot x = a \cdot x + b \cdot x}$$

8. **Use Proper Notation** There are many notational conventions in mathematics. You need to follow the accepted conventions when using notation. For example, A summation or integral symbol *always* needs something to act on. The expressions

$$\sum_{i=0}^{n} \qquad\qquad \int_a^b$$

by themselves are meaningless. The expressions

$$\sum_{i=0}^{n} a_n \qquad\qquad \int_a^b f(x)\mathrm{dx}$$

have well-understood meanings.

As another example,

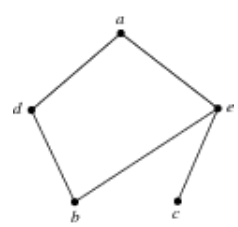
$$\lim_{h\to 0} = \frac{2x+h}{2} = \frac{2x}{2} = x$$

is incorrect. It should be written

$$\lim_{h\to 0} \frac{2x+h}{2} = \frac{2x}{2} = x$$

9. **Be Precise and Specific** The more precise and specific you are, the easier it is for the reader to understand what you are writing. In addition, the extra details are necessary for the reader to believe your claims. You have thought about the problem for a while so some steps may seem obvious to you. However, without the omitted details, it may not be obvious to the reader.

For example, suppose you assert that the next graph is bipartite (see Definition 10.10).



Writing

> The graph is bipartite because we can partition the vertices into two disjoint sets, V_α and V_β.

is not specific enough. Instead, you should write

> The graph is bipartite because we can partition the vertices into two disjoint sets, $V_\alpha = \{a, b, c\}$ and $V_\beta = \{d, e\}$.

Listing the vertices in the sets helps the reader to verify your claim with less effort. It also makes it clear that you really understand what you just wrote.

10. **Parentheses Are Important** A convention in summation notation is that the summation applies to the *term* immediately following the summation sign. Thus

$$\sum_{k=1}^{n} 2k+3 = \left(\sum_{k=1}^{n} 2k\right) + 3 = 2\frac{n(n+1)}{2} + 3 = n^2 + n + 3$$

but

$$\sum_{k=1}^{n} (2k+3) = \sum_{k=1}^{n} 2k + \sum_{k=1}^{n} 3 = 2\frac{n(n+1)}{2} + 3n = n^2 + 4n$$

11. **Do not omit necessary words** Suppose you are forming a committee of size c from a group of m people. The following statements all omit necessary words:

Statement 1

There is a group m and they must choose c people from it which gives $\binom{m}{c}$.

This should be written:

There is a group *of m people* and they must choose c people from the group. There are $\binom{m}{c}$ *ways to do this*.

Statement 2

We have m people to choose from and we are choosing c people $\binom{m}{c}$.

This should be written:

We have m people to choose from and we are choosing c people *so there are* $\binom{m}{c}$ *ways to form the committee*.

Statement 3

Pick the group of c people from m. This is done by $\binom{m}{c}$.

This should be written:

Pick the group of c people from m. This *can be done in* $\binom{m}{c}$ *ways*.

Statement 4

m = people c = committee

This should be written:

m = *the number of* people c = *the size of the* committee

12. **Don't change the meaning of the notation in midstream** Suppose you are forming a committee of size c from a group of m people and also must appoint a chairperson from among the c committee members. The following statements change the meaning of the notation somewhere in the middle.

Statement 1

The group of m people first choose a chairperson 'c' from their group.

The letter 'c' should not be used to designate the chairperson since it has already been defined to be the *number* of people on the committee.

Statement 2

There is a group m that need to select a committee of size c and appoint a chairperson.

The group has *size m* — it is not *named m*.

13. **The words** amount **and** number **are not interchangeable** The word *number* is used with things that can be counted. The word *amount* is used with things that cannot be counted.

Examples

There are a number of students in the class who forgot to memorize the logarithm properties.

I have forgotten the proper amount of water to add to this recipe.

The number of classes I am taking this semester has decreased.

The amount of work required by my professors has increased every semester.

incorrect The amount of people on the committee hasn't been decided.

correct The number of people on the committee hasn't been decided.