The *Visualizing Combinatorial Proofs* Project

A well-known proverb says, "*Tell me and I'll forget. Show me and I may not remember. Involve me and I'll understand.*" While this project does contain a bit of "telling" and "showing," it was created with you in mind in the hope that your involvement will lead you to a better understanding of combinatorial proofs.

VCP (Visualizing Combinatorial Proofs) is a collection of modules designed to help undergraduate math students work through a variety of combinatorial proofs using examples. Each module is based on a particular identity and its combinatorial proof. Modules contain two pieces: an exposition and a visualization which are explained in more detail below. The modules were created at the level of a second year mathematics student, though others may find them interesting and useful.

1 Exposition

1.1 Motivation

The motivation section presents the background for the identity. It discusses the significance of the identity and the people who were involved in its development. Too often mathematical topics are introduced apart from their historical setting, yet knowing the context for a particular identity gives it more meaning. The motivation section is not only helpful but also intriguing. You may find that the background of the proof is sometimes just as fascinating as the proof itself.

1.2 Preliminary Ideas

This section presents and develops definitions necessary for understanding that particular exposition. It also alerts you to other more basic definitions that you will need. Reading these definitions carefully will help you understand the rest of the exposition.

1.3 The Problem Presented

This section presents the identity in the form of a theorem.

1.4 Solution by Counting

The solution by counting section gives a combinatorial proof of the identity presented. It is important that you actively read the proof (as opposed to speed-reading, skimming, or skipping the proof in favor of the visualization). Unlike many proofs presented in textbooks, the combinatorial proofs within these expositions include a lot of details and intermediate steps. It would be worth your time to take advantage of the step-by-step explanation and critically think about what you are reading.

1.5 Visual Example

The exposition finishes with a specific example that will be used to illustrate the ideas of the proof. That example is set-up and explained in this section.

After opening the exposition, the user is presented with the option to start the visualization.

2 Visualization

The visualization is a critical element of this project. With the exception of Oresme's sequence, the visualization presents a pictorial example and not the actual proof. In most cases, it is nearly impossible to accurately present a proof purely pictorially. Words are necessary in laying out the ideas of a proof. However, it is often a picture or a concrete example that makes the words "click." Viewing the visualization without giving thought to the proof is useless, but using the visualization to complement the exposition will enhance your understanding.

2.1 Using the Visualization

The theorem that corresponds to the visualization is shown within the visualization. It is displayed either at the top of the screen or labelled elsewhere.

The visualization is begun by clicking the **Start Visualization** button found in the bottom right-hand corner. Each visualization contains explanatory text that serves to clarify what is happening in the visualization. After you have read the text and understand what is taking place on the screen, you may click **Continue** to take you to the next piece of the visualization. You also have the option of controlling the speed of the visualization. By using the slider on the bottom left of the screen, you can alter the pace of the animation at any time from slow to fast. If you would like to go through the visualization step-by-step, click the **Pause** button. You can then **Step** through that piece of the visualization. Note that you can "pause" the visualization before beginning, which allows you to step through from the beginning. To resume animation, click the **Play** button. Once the visualization is complete, the **Continue** button says "Done," and the other buttons are greyed out.

The visualizations also contain a toolbar on the left side of the screen that allows you to easily navigate through the modules. There is an option to bring up the exposition if you have closed it and to view the "Common Definitions" file should you need it. You can restart the current visualization if you so desire or you can load another module.